Matching Auctions for Cut Blocks

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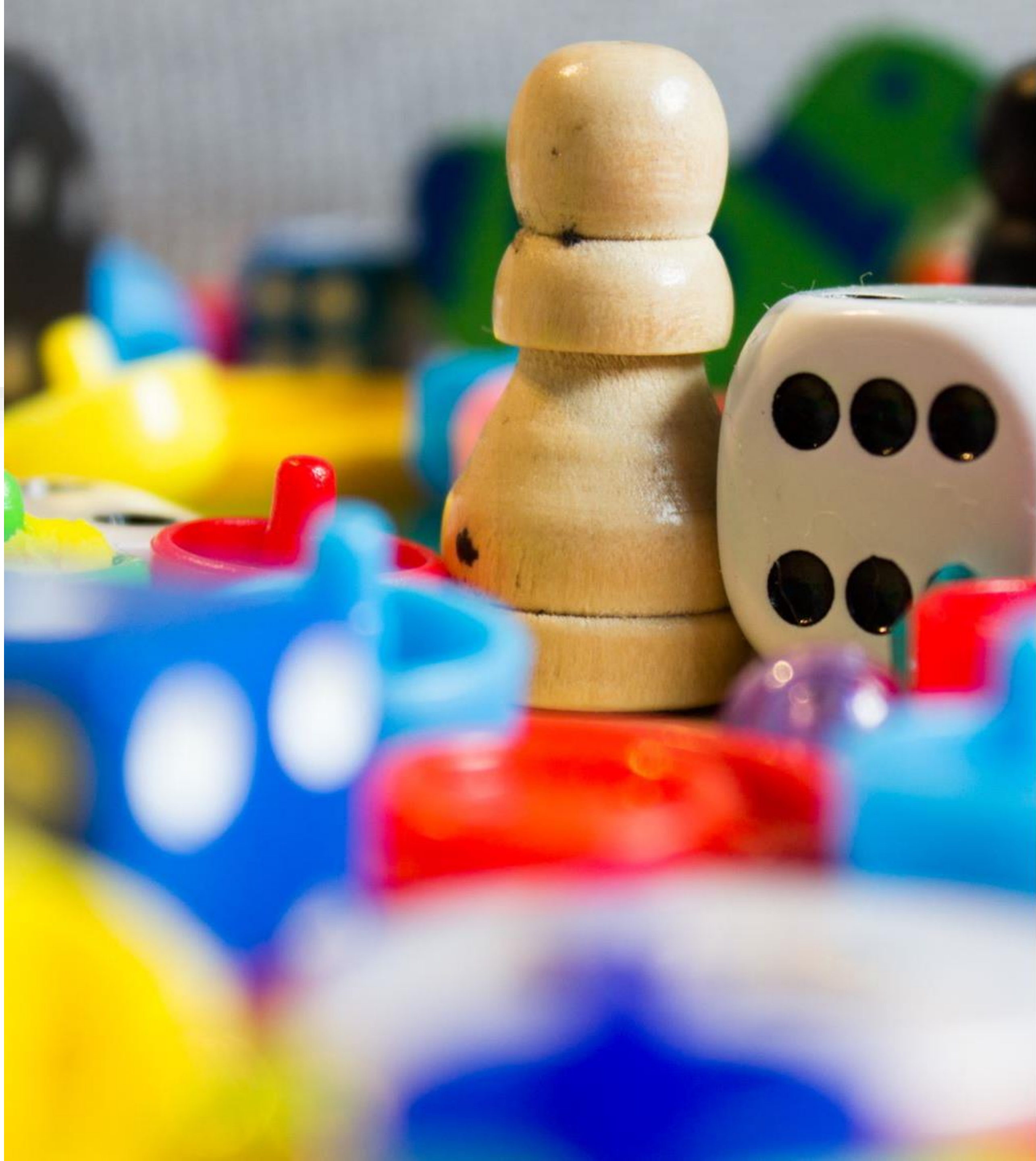
Objective

- Introduce matching auctions on graphs as a way of allocating timber
- An alternative way of ‘getting the right log to the right mill’
- With sufficient digital infrastructure could be extended to log sorts, even while tree is on the stump

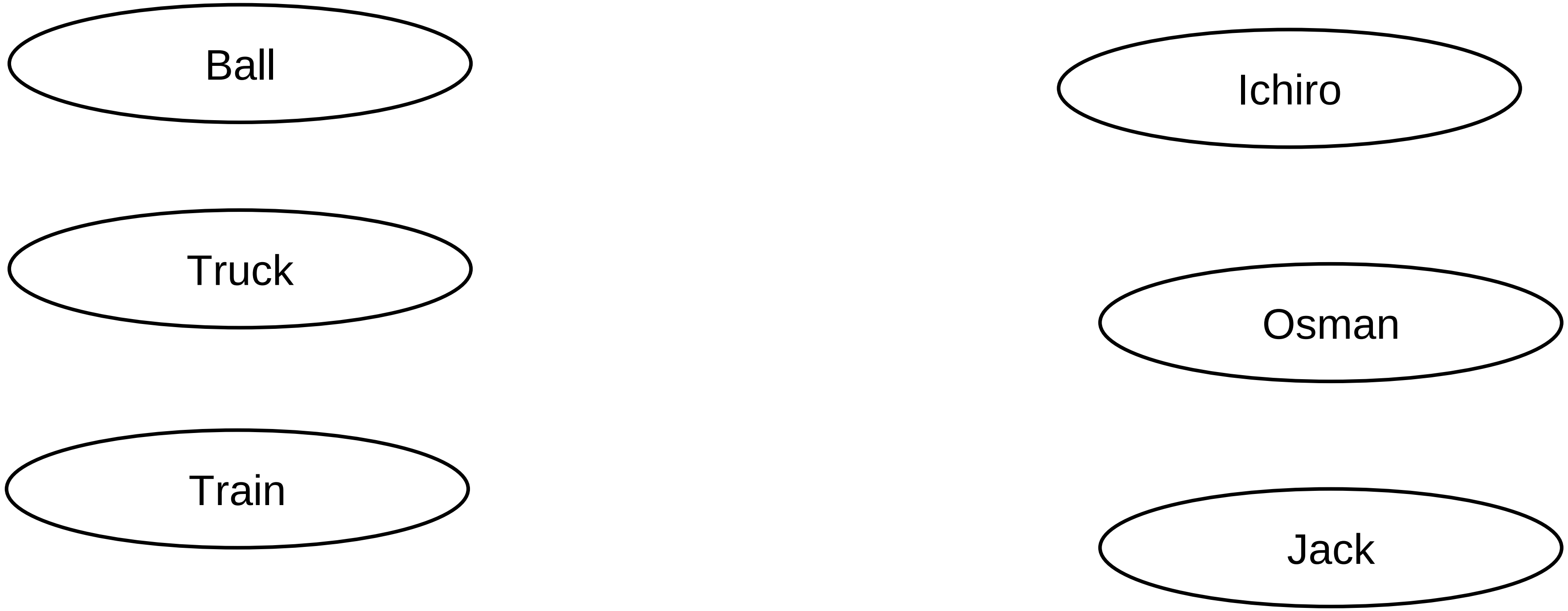
Intro to Matching Theory

- Suppose you find yourself in a situation where you are responsible for three children and you have to keep them happy for a number of hours with a minimum of fuss. You have available to you three toys. How do you distribute the toys among the children to make them as happy as possible?*

*From David F Manlove. Algorithmics of matching under preferences.



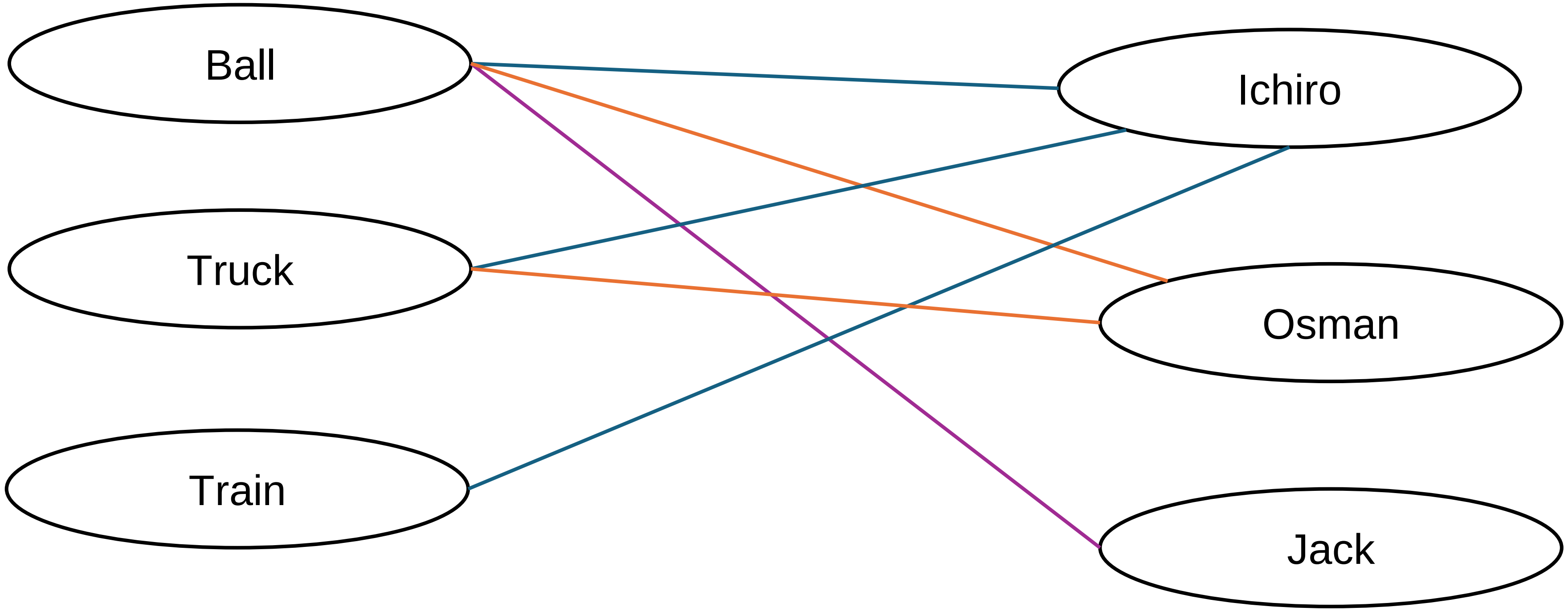
Model the problem as a bi-partite graph



Bi-partite because there are two nonempty, nonoverlapping sets (A and B) where every edge has a starting point in A and an end-point in B

Model the problem as a bi-partite graph

The edges represent a match between a child and a toy.



Bi-partite because there are two nonempty, nonoverlapping sets (A and B) where every edge has a starting point in A and an end-point in B

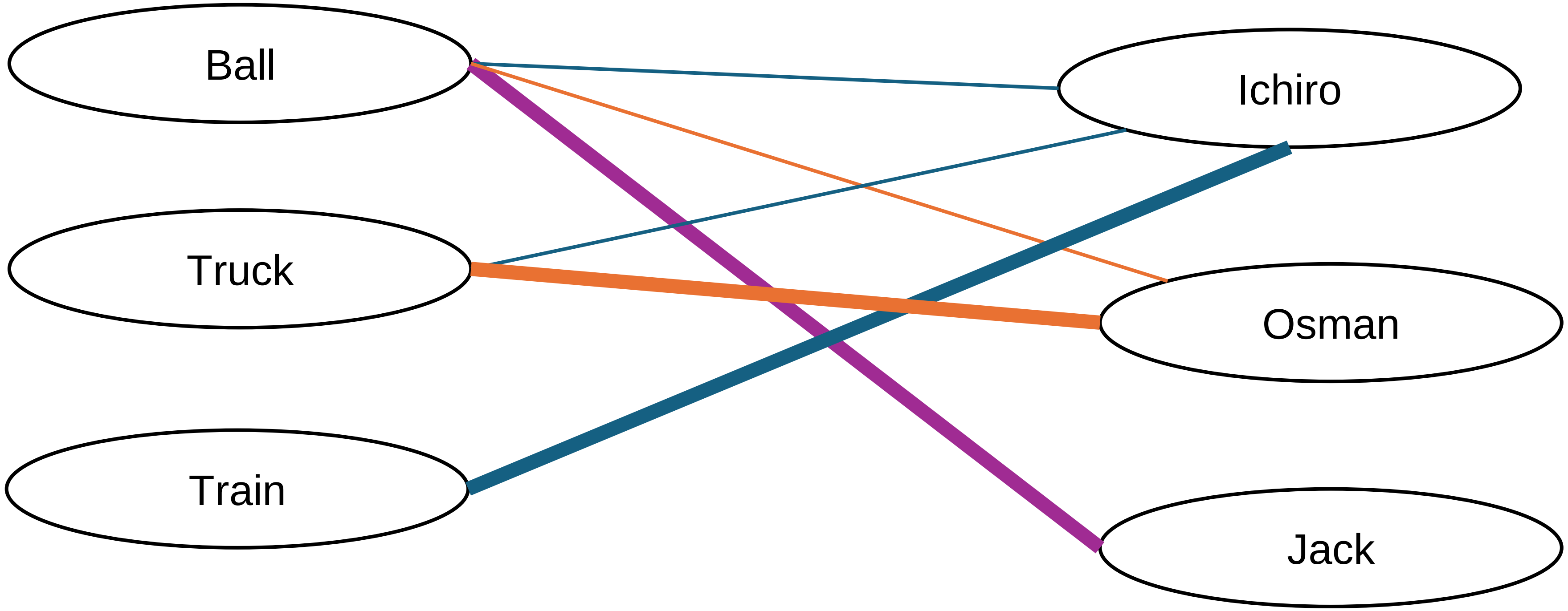
Graph can be represented as a matrix

$$\begin{array}{c} \\ Ichiro \\ Osman \\ Jack \end{array} \begin{array}{c} \begin{array}{ccc} Ball & Stick & Box \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

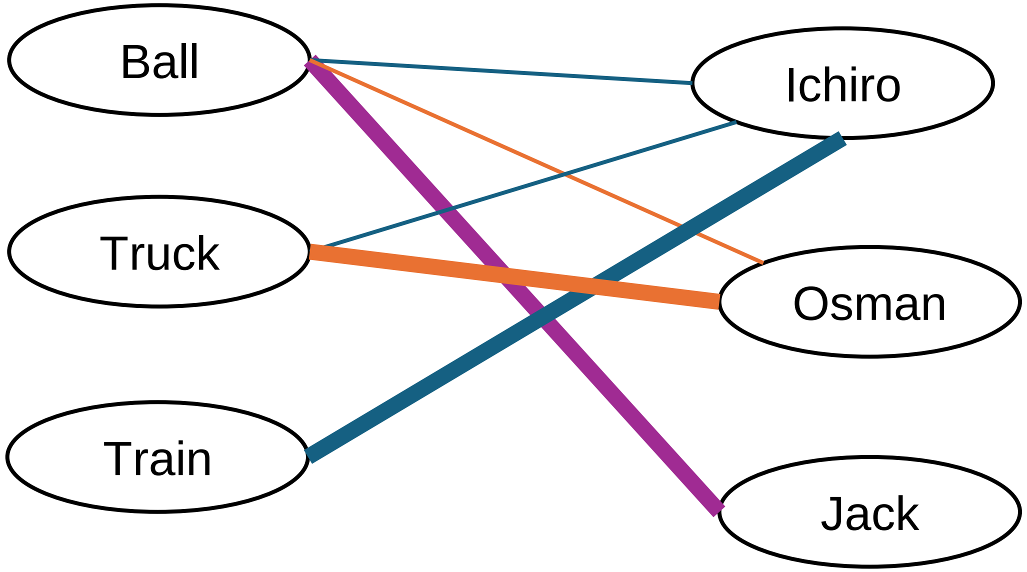
Goal: Perfect matching

- A perfect match in a graph occurs when every node on the right is associated with a node on the left AND no node on the left is assigned to more than one node on the right.
    - That is, every child has a toy and no children are left fighting over a single toy.

A perfect match



A perfect match



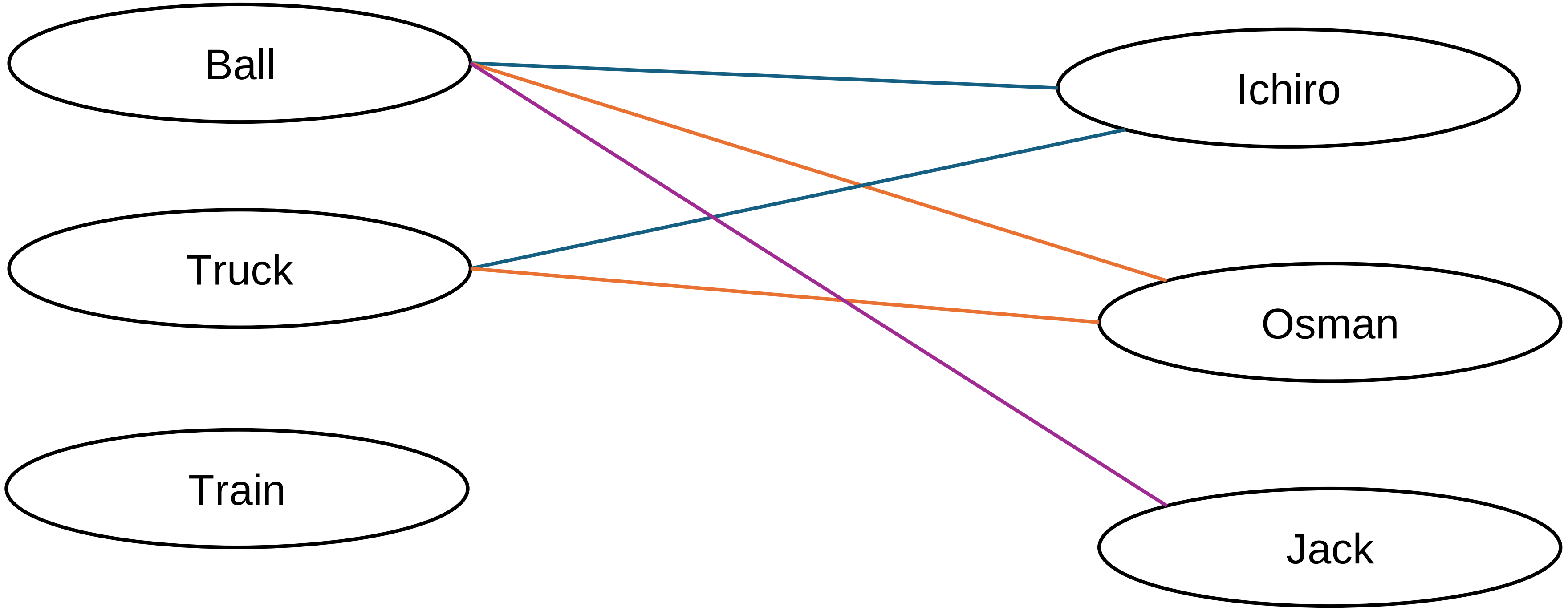
$$\begin{array}{c} \\ Ichiro \\ Osman \\ Jack \end{array} \begin{array}{c} \begin{matrix} Ball & Stick & Box \end{matrix} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

Constricted Sets

- Perfect matching are not always possible
- Suppose you take any set of nodes on the right-hand side of the graph and call it S*. The Neighbors of S, called N (S) is the set of nodes that have a connection to any member of S. If the number of nodes in S is larger than the number of nodes in N (S) then we say that S is a constricted set. In our example of toys and children this would mean that children's toy preferences are such that there is a fight over a toy (or potentially more than one toy if there are enough children and enough toys)

*In the example S could be any set of children with two or more members {{Ichiro, Osman}, {Osman, Jack}, {Jack, Ichiro}, {Ichiro, Osman, Jack}}

Constricted Set



Ichiro, Osman and Jack form the set, *S.* The neighbor set, *N(S)* is just the Ball and Stick. The size of *S*> *N(S).*
There is going to be a fight over the toys and we have a constricted set.

Prices and Constricted Sets

- When we don’t have a preference ordering we can’t do much with the constricted set
- Prices us allow to solve this problem…

Multiple Good Auctions on Bipartite Graphs: A Forestry Example

- Let there be four mills m_i and four cut blocks b_j - $i, j \in \{1..4\}$
- Each mill has a valuation of a cut block $v_{i,j}$
- The seller of the cutblock offers them at price p_j
- Payoff to a mill is $v_{i,j} - p_j$
- Sellers of harvest rights that maximize the payoff to a mill are the *preferred sellers* of mill *i*
- If payoffs are all negative for a mill, it has no preferred seller.

Multiple Good Auctions on Bipartite Graphs:
A Forestry Example

- A set of market clearing prices p_j^*:
  - Will award a cut block to each mill
  - Each block will go to the mill that values it most
  - This results in a *perfect matching* and it maximizes the possible sum of payoffs to all sellers and buyers

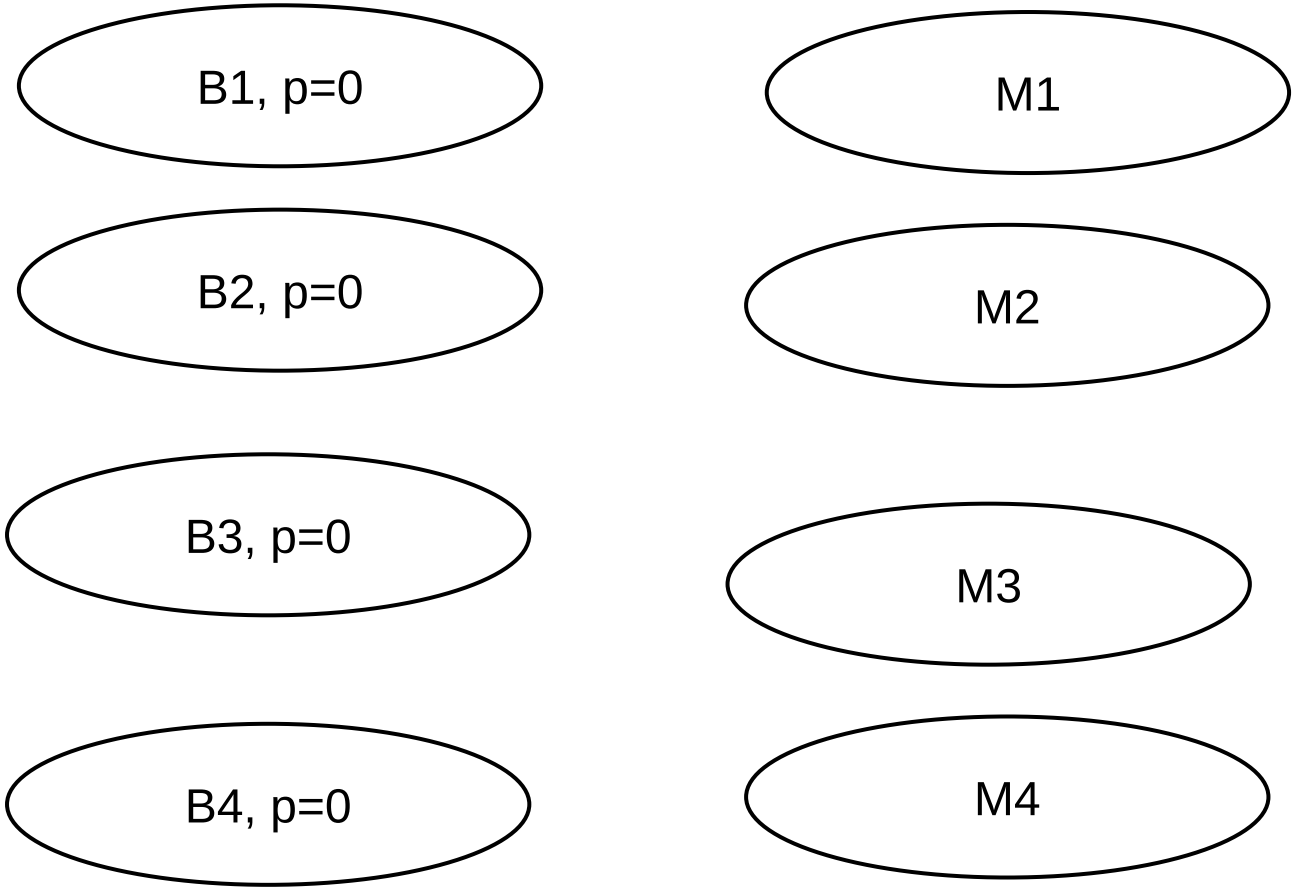
Finding the Market Clearing Price

- One method is an Ascending Auction*

1. Set prices to zero
2. Mills check what preferred block is at that price
3. This block, or several blocks are a match for each mill if there are more than one
4. If there is a perfect match, then the price is market clearing.
5. If there is no perfect match, for all sets, S, of the mills, check if there is a constrained set among the cut blocks N(S)
6. If there is a constrained set, all members of N(S) increase their price by $1
7. If all prices exceed zero, subtract the excess from all prices.

*Gabrielle Demange, David Gale, and Marilda Sotomayor. “Multi-Item Auctions”. In: Journal of Political Economy 94.4 (Aug. 1986), pp. 863–872.

Finding the market clearing price



$v_{1,j} = \{3,2,1,0\}$

$v_{1,j} = \{2,2,1,0\}$

$v_{1,j} = \{4,2,1,0\}$

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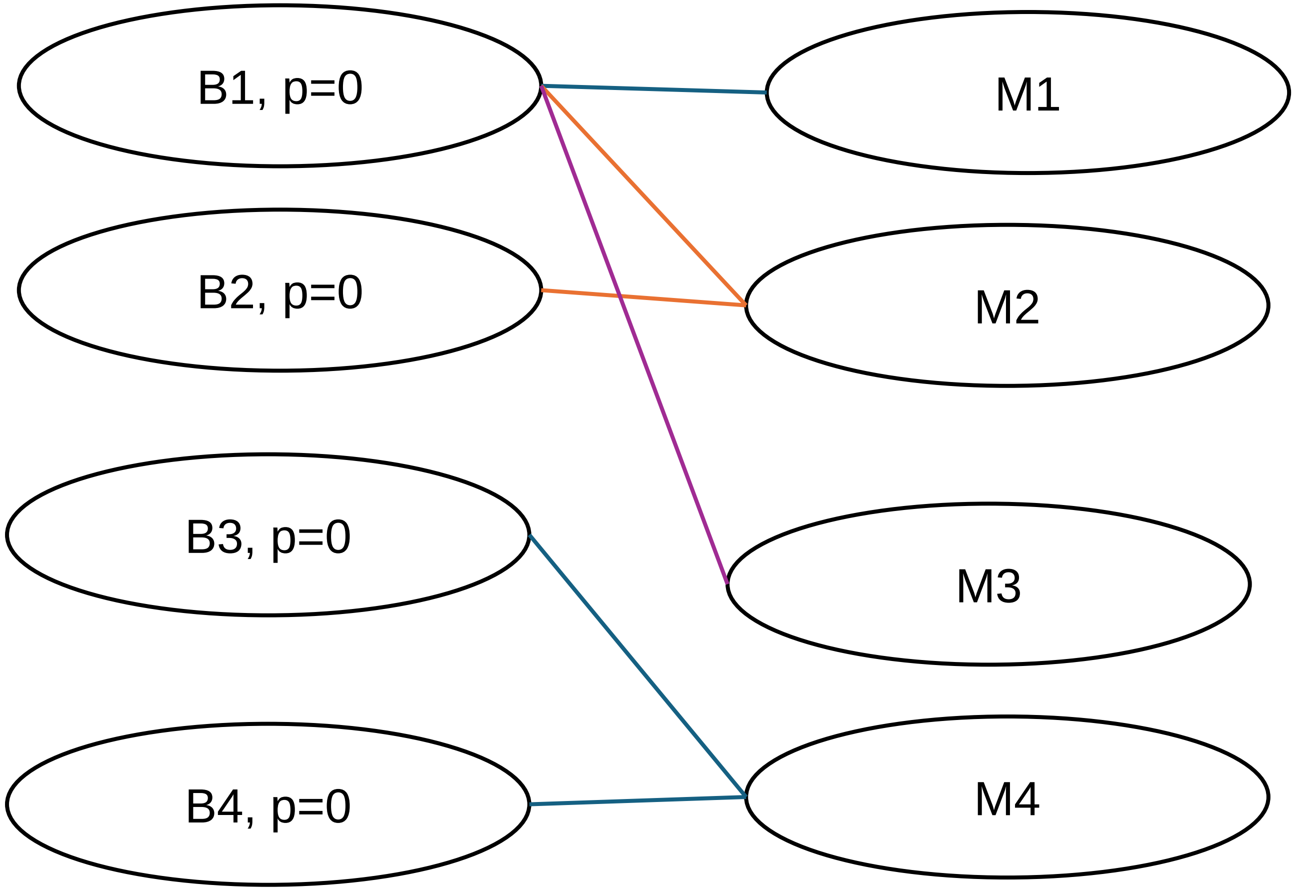
Finding the market clearing price



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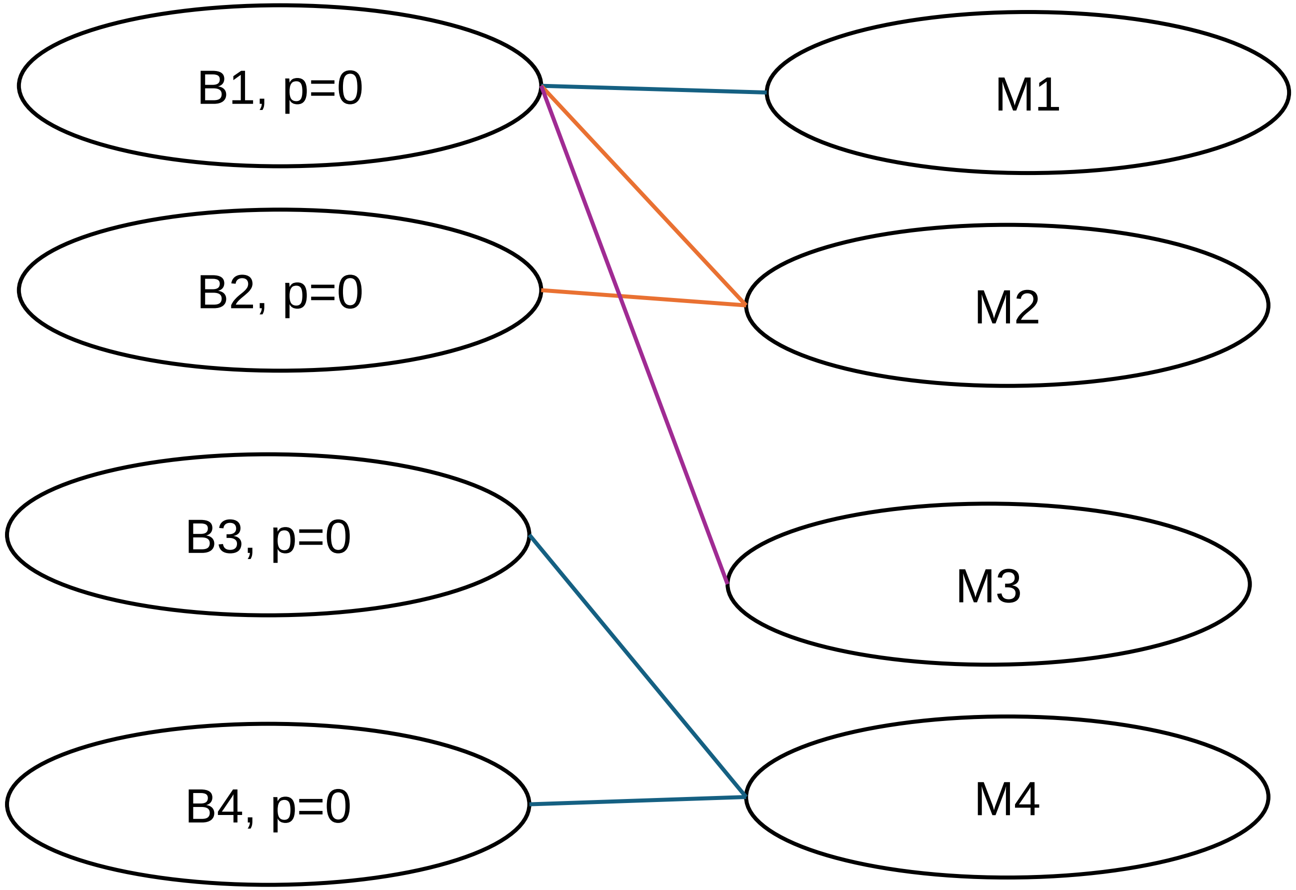

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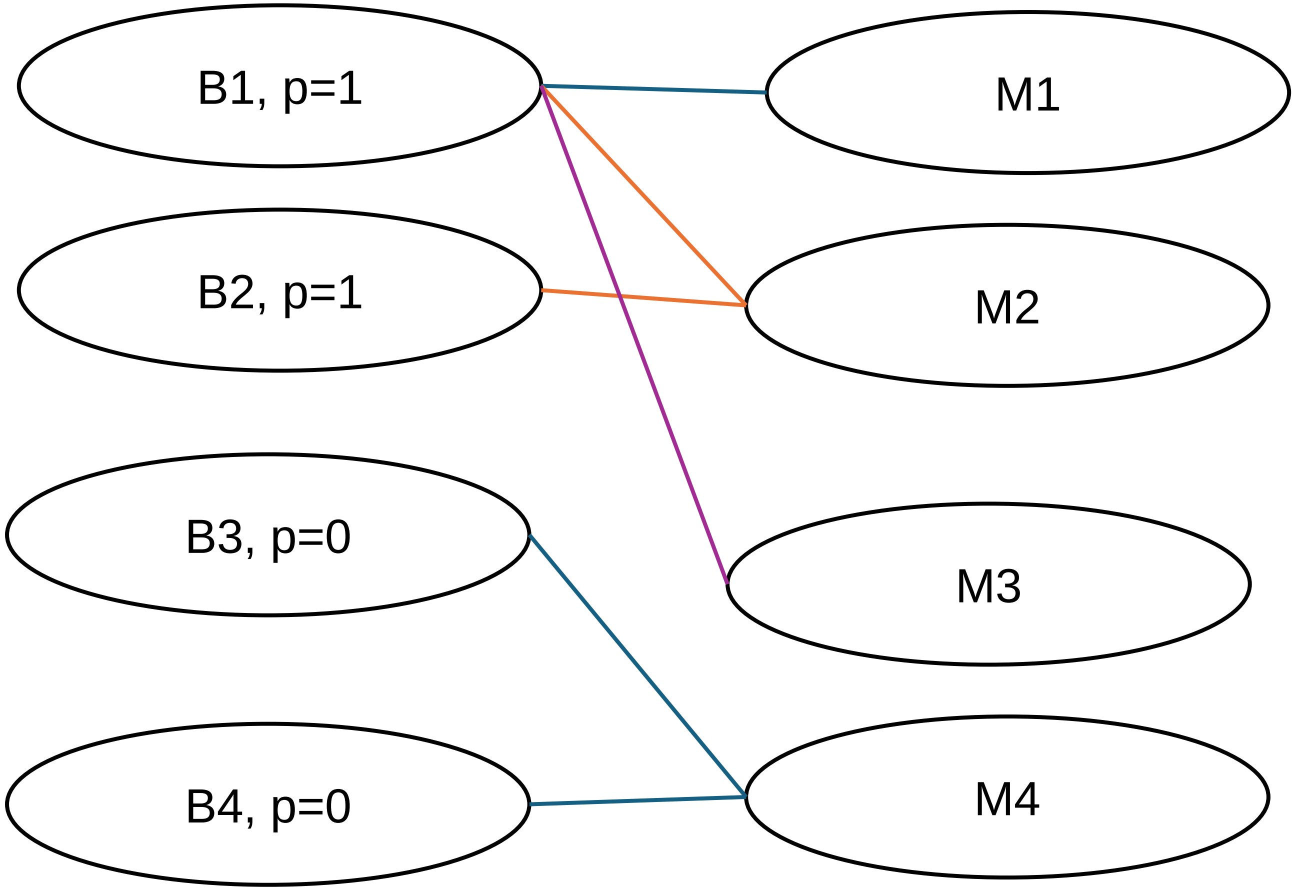
Size of S={M1,M2,M3} > N(S)={B1,B2} so we have a constrained set.

Finding the market clearing price



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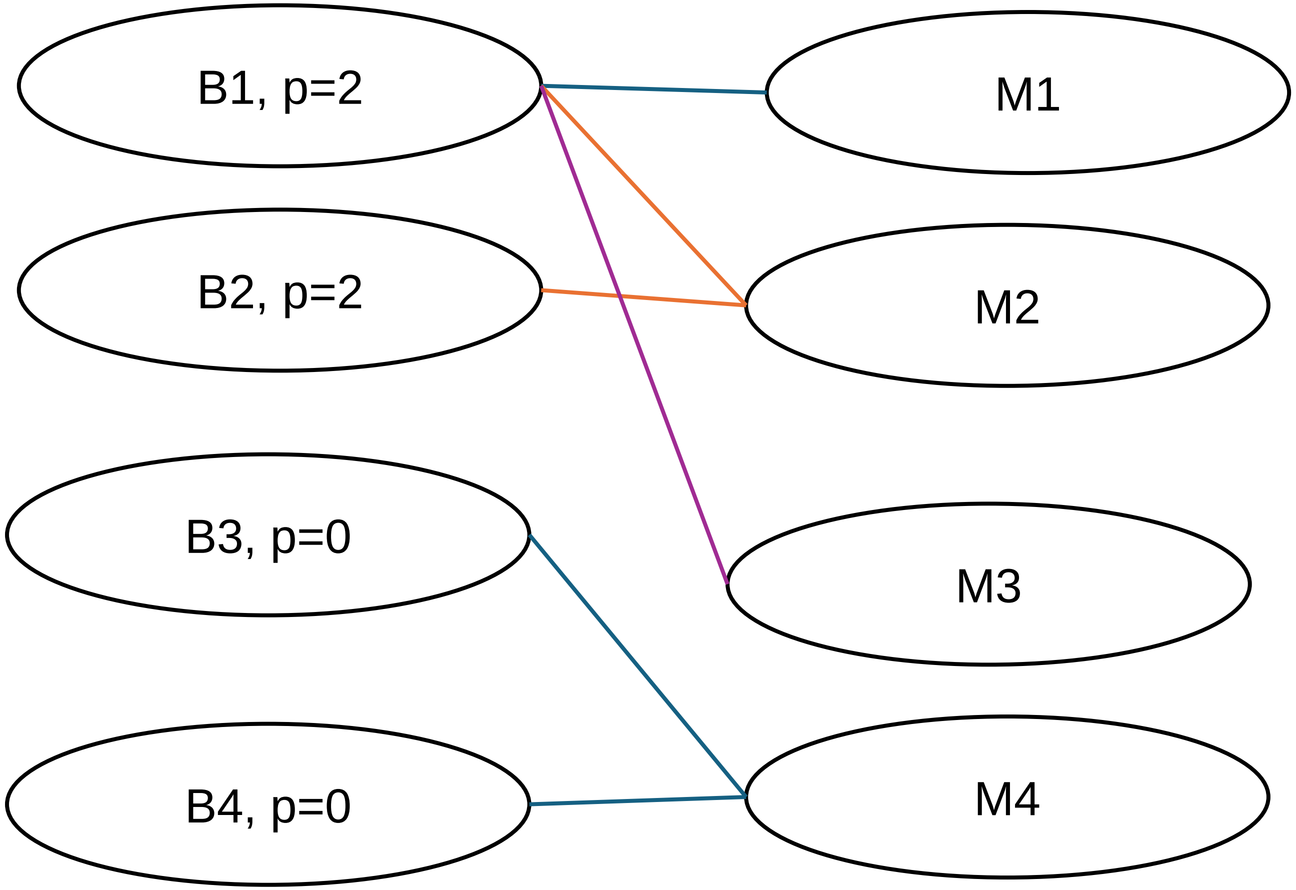
Size of S={M1,M2,M3} > N(S)={B1,B2} so we have a constrained set.
Increase price by $1 of N(S)

Finding the market clearing price



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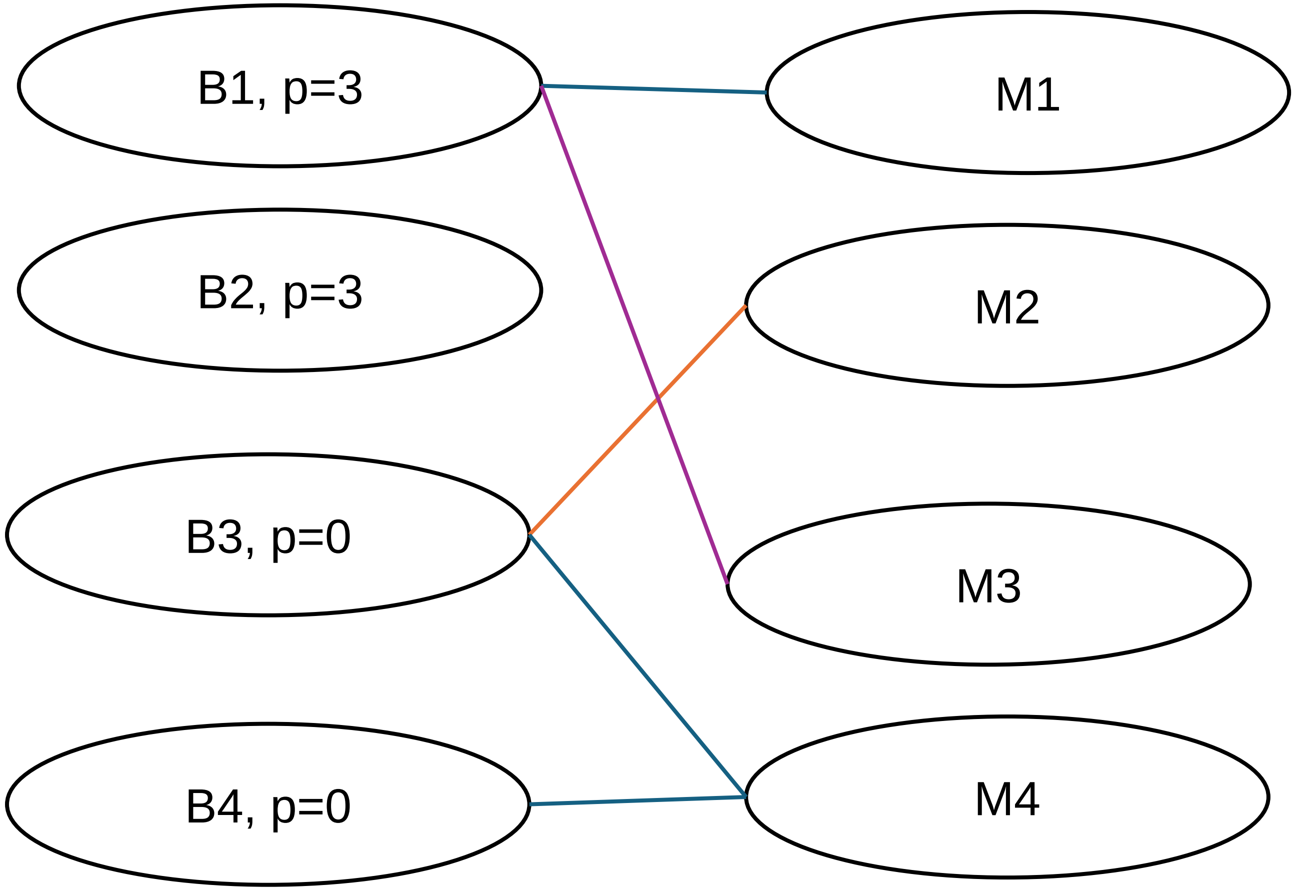
Still the same, so we increase by $1 again

Finding the market clearing price



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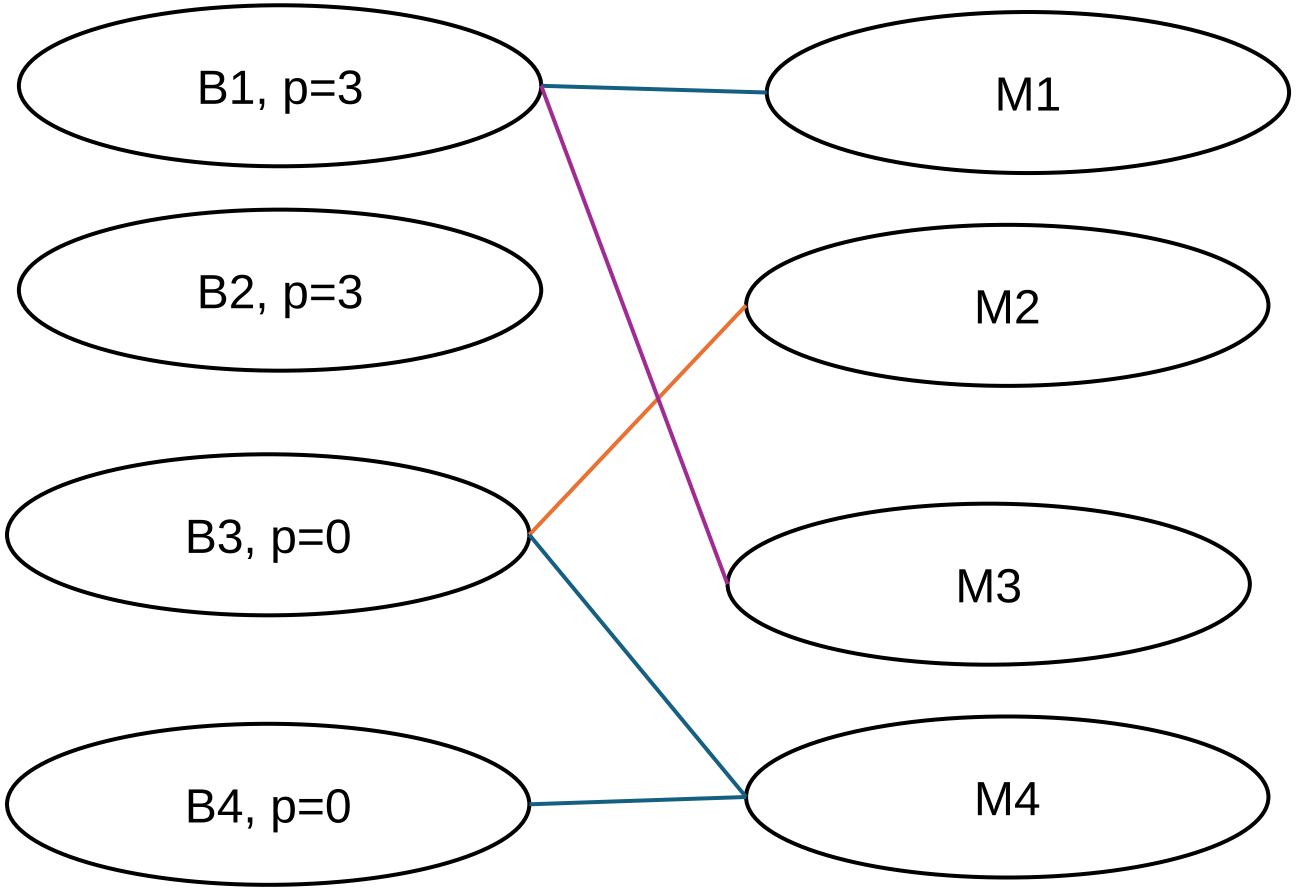
Still the same, so we increase by $1 again (twice)

Finding the market clearing price



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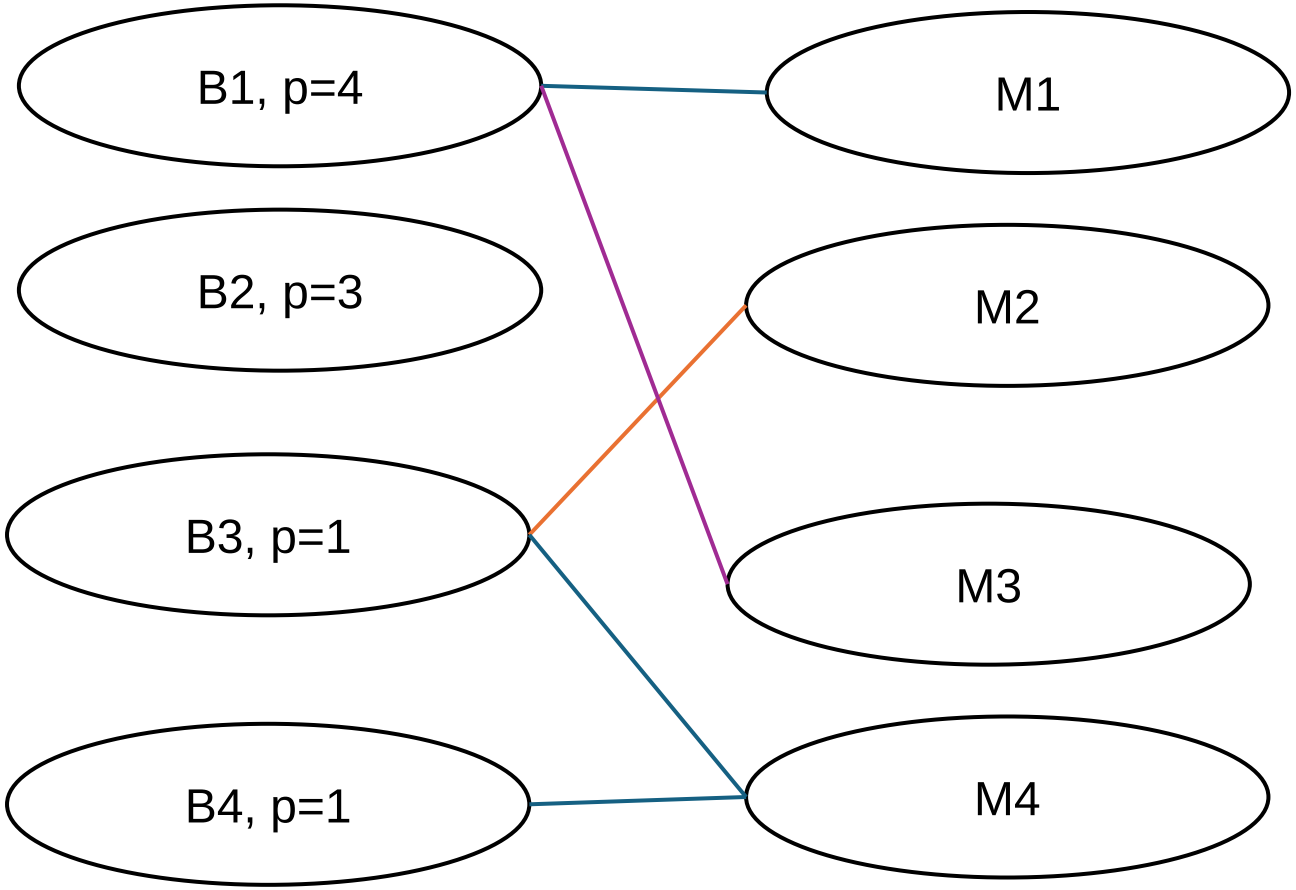
$S_1 = \{M1, M3\}$ *and* $S_2 = \{M2, M3\}$ *are both constrained.*

Finding the market clearing price



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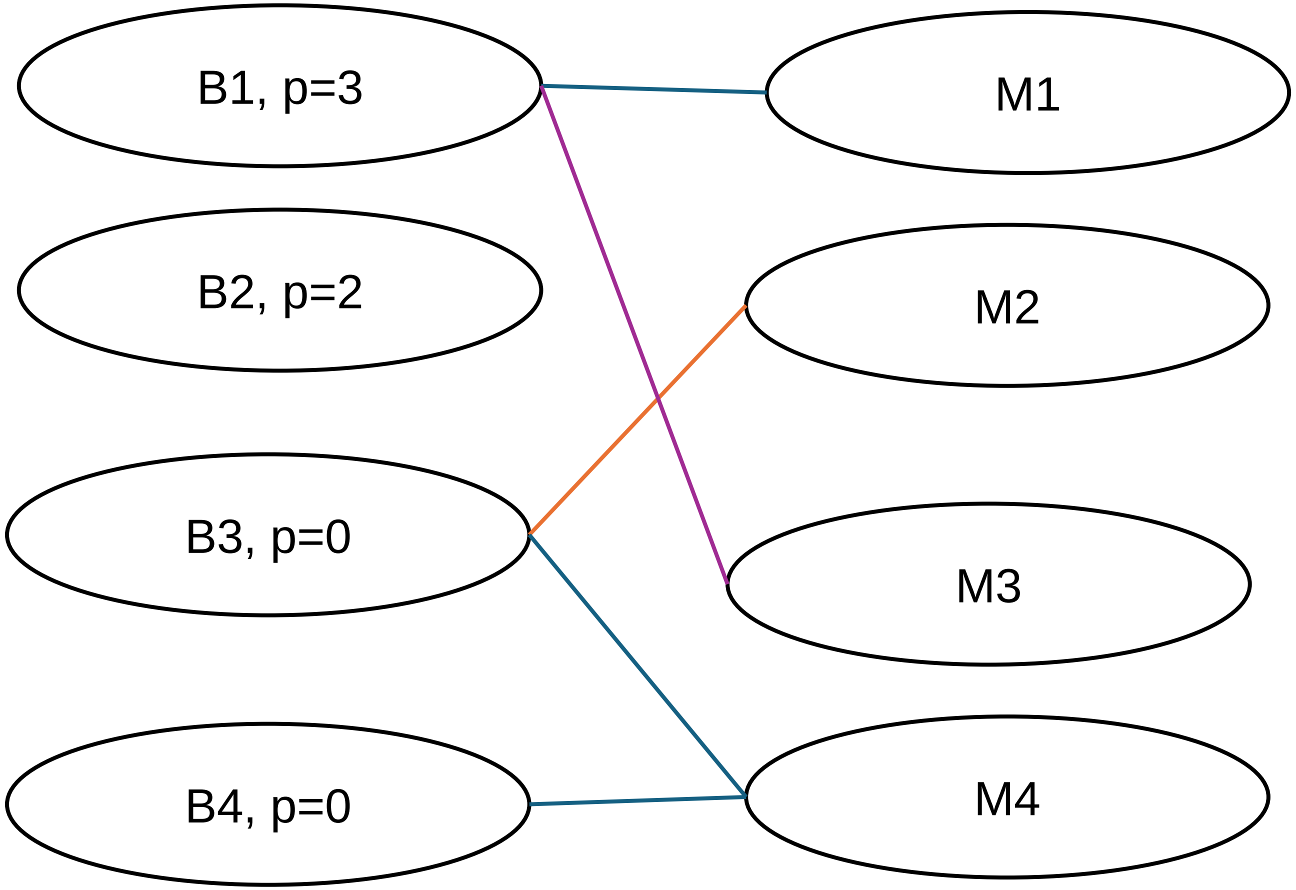

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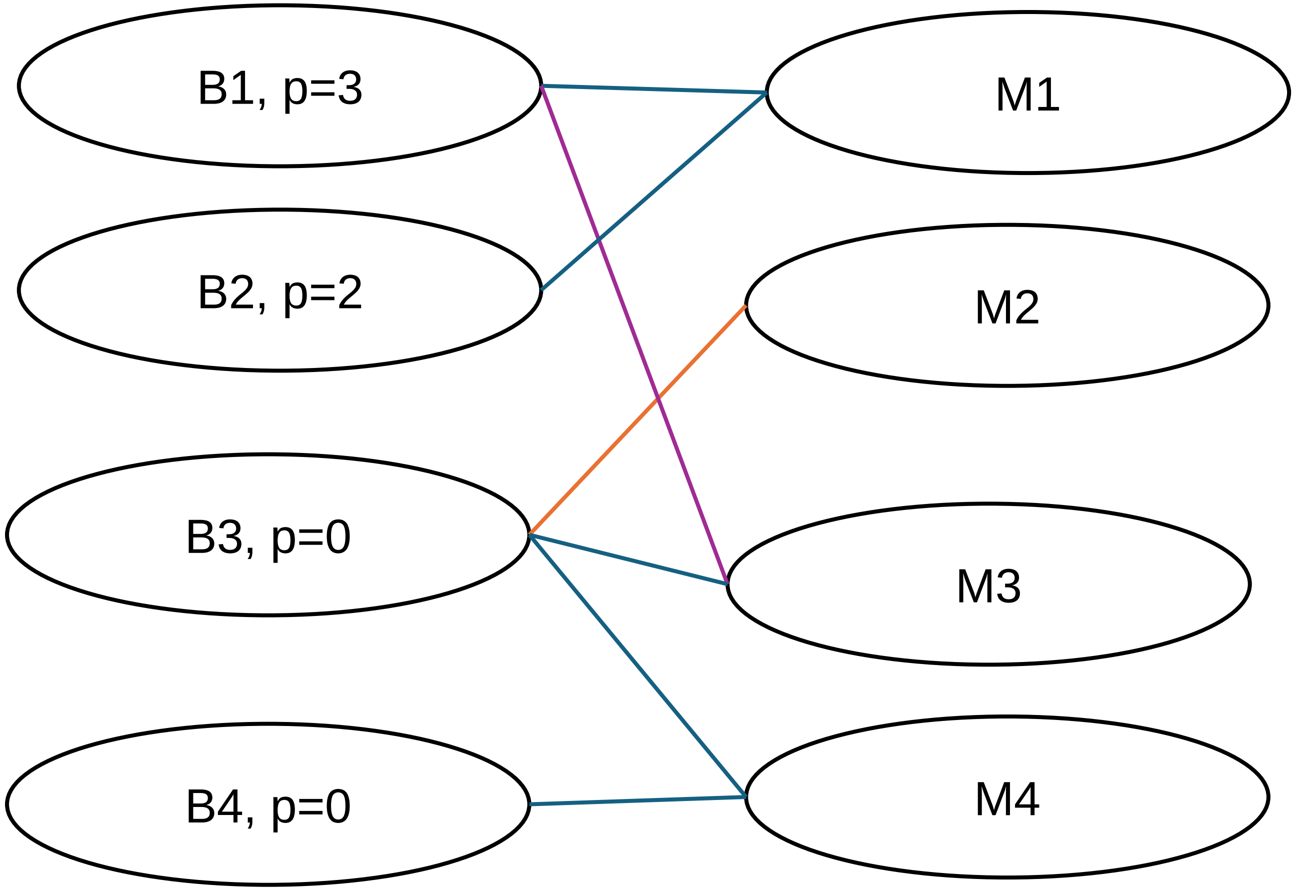
Renormalize prices.

Finding the market clearing price



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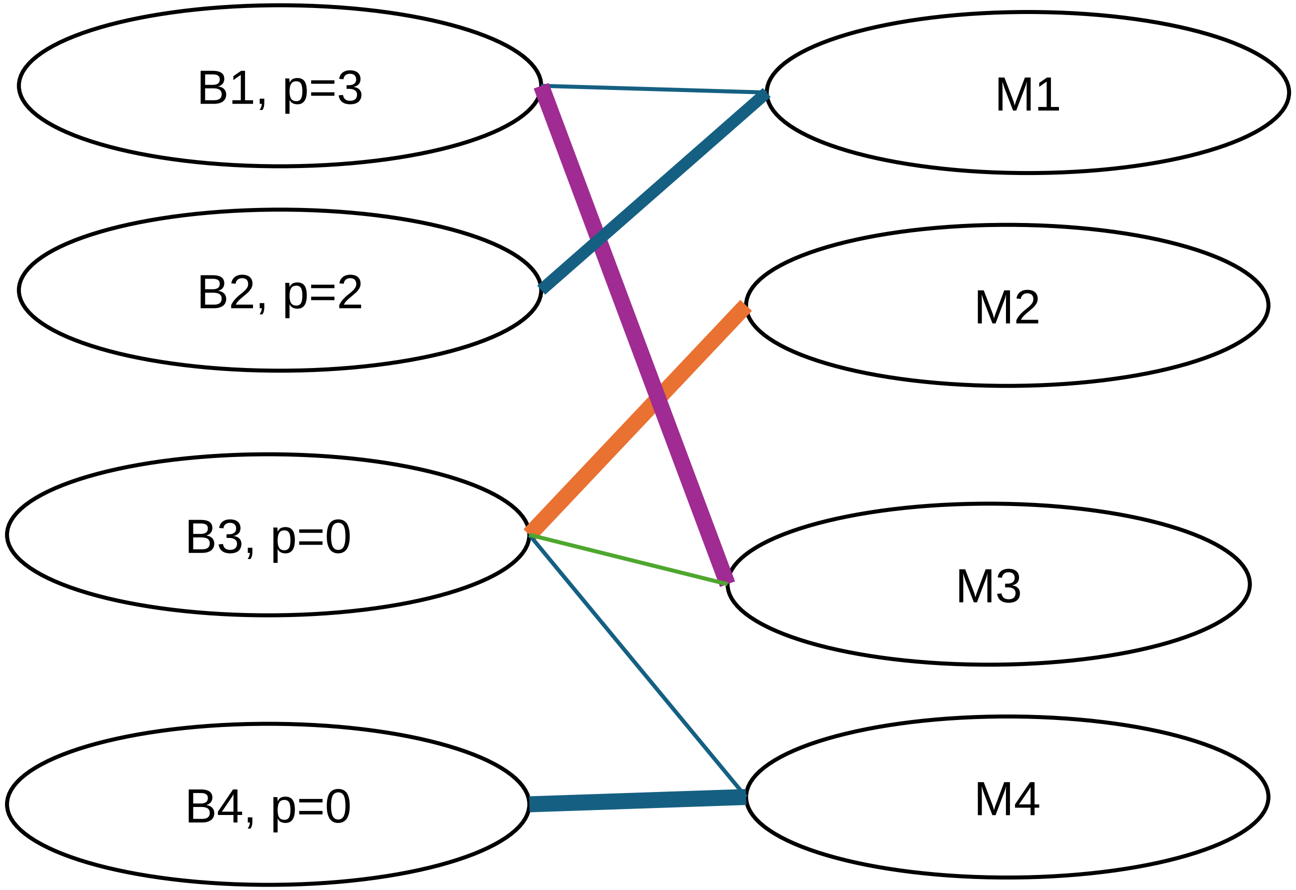

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And we have a perfect match $M3 \leftrightarrow B1, M1 \leftrightarrow B2, M2 \leftrightarrow B3, M4 \leftrightarrow B4$!

Optimality

- *Optimality:* For any set of market-clearing prices, a perfect matching gives the maximum sum of valuations.

How can this be extended

- For mills greater than cut blocks – null blocks with zero value are added for the algorithm to solve
- If blocks exceed mills, null mills are added.
- If mills need to take multiple blocks to get necessary volume for the year can submit under multiple ids.

Why would you want to do this?

- If there are limited numbers of mills in an area and competition is a concern, can use this method to get closer to optimal pricing
- Auction ALL of the blocks for the year all at once. Much harder to manipulate pricing.
- Potential for matching species/grade to buyer, potential for greater price discrimination and efficiency in weakly competitive markets.

Questions?

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